Flow Structure and Turbulence in the Inner Part of Turbulent Boundary Layers over Super-Hydrophobic Surfaces

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ABSTRACT

Digital holographic microscopy (DHM) and particle image velocimetry (PIV) are used to characterize the flow structure and turbulence in the inner part of turbulent boundary layers over several super-hydrophobic surfaces (SHSs). These SHSs are generated using various methods, including spraying, etching and sandblasting. The measurements have been performed at friction Reynolds numbers ($Re_f$) varying from 700 to 4400, and normalized rms values of roughness heights ($k_m$) in the 0.2 to 3.3 range. The slip velocity and local wall friction are calculated directly from the mean velocity and total stress at the top of the SHS roughness. In addition to calculating profiles of mean velocity, Reynolds and total stresses, data analysis examines the distributions of eddy viscosity, Prandtl mixing length, shear production and turbulence spectra. For the SHSs with randomly distributed roughness, results show that the drag reduction diminishes with increasing $k_m$ and $Re_f$. For $k_m$<1 and relatively low $Re_f$<800, the viscous stress is lower than that of the smooth wall and the Reynolds shear stress is nearly zero, resulting in 10 to 30% reduction of wall friction. The slip velocity varies from 15 to 30% of the freestream mean flow speed, and the slip length $\lambda$ falls between 30 to 110μm ($3<\lambda<10$). For $k_m$=3.3 and relatively high $Re_f$<4400, the near wall stress is dominated by Reynolds shear stress and the drag is higher than that of a smooth wall. For the drag reducing SHSs, the peaks values of Reynolds shear stress are lower than that of the smooth wall, resulting in a lower turbulence kinetic energy (TKE) shear productions in the log region. In contrast, for the drag increase SHSs, the log region has higher Reynolds shear stresses and TKE production. Very close to the wall ($y^+<5$), all the SHSs have higher velocity fluctuations, TKE production, eddy viscosity, and Prandtl’s mixing length compared to those of the smooth wall. However, in the log region, the eddy viscosity and Prandtl’s mixing length profiles of the SHSs coincide to those of the smooth wall. Streamwise spatial energy spectra of both streamwise and wall-normal velocity fluctuations show that drag-reducing SHSs alter the structure of near-wall turbulence. They reduce the inertial range streamwise fluctuations, and increase the wall normal large scale fluctuations. The latter effect diminishes with increasing distance from the wall, but the former trend persists.

INTRODUCTION

Reduction of the hydrodynamic skin-friction is desirable in many engineering applications, such as marine vessels, long pipelines, etc. One possible method is to functionalize the surface with super-hydrophobicity, a property attributed to a combination micro/nano-scale roughness and hydrophobic chemistry (Liu et al. 2013). When submerged in water, the super-hydrophobic surface (SHS) promotes a Cassie-Baxter state, consisting of air pockets (plastrons) trapped between roughness elements (Rothstein 2010). The lower viscosity air trapped beneath the higher viscosity liquid is expected to cause skin-friction reduction.

It has been demonstrated and widely accepted that SHSs reduce drag in laminar boundary layers, starting from the early demonstrations by Cottin-Bizonne et al. (2003), Ou et al. (2004) and others. Based on Navier’s model (Rothstein 2010), the slip boundary condition is characterized by the so-called slip velocity $u_s$ and the slip length $\lambda$ satisfying

$$u_s = \frac{\tau_w}{\rho v^2}$$

where $\tau_w$ is the wall shear stress, $\rho$ is the fluid density, and $v$ is the free stream velocity. For a smooth wall, $u_s = 0$ and $\lambda = 0$. However, for SHSs, $u_s$ and $\lambda$ can be significantly larger, reducing the skin-friction.

$$\lambda = \frac{1}{\lambda}$$

where $\lambda$ is the so-called slip length and $\lambda$ is the so-called slip velocity. For a smooth wall, $\lambda = 0$ and $\lambda = 0$. However, for SHSs, $\lambda$ and $\lambda$ can be significantly larger, reducing the skin-friction.
\[ u_\tau = \lambda du / dy, \] where \( y \) is the wall-normal direction, and \( u \) is the streamwise velocity. Experiments performed in various microfluidic devices have reported that the values of \( \lambda \) on SHS extend to 100 \( \mu m \), and the drag reduction to 40\% (Lee et al. 2008, Ou & Rothstein 2005, Song et al. 2014, Srinivasan et al. 2013). Key geometric parameters of SHSs associated with drag reduction are the solid fraction \( \Phi_s \) and pattern wavelength \( \beta \). Ybert et al. (2007) show and Lee et al. (2008) confirm that \( \lambda - \beta \Phi_s^{0.5} \). Theoretical works by Lauga & Stone (2003) reveals a functional relation \( DR=f(\Phi_s, \beta/L_s) \), where \( DR \) is the drag reduction, and \( L_s \) is the characteristic length scale of the laminar flow. This relation is presumably independent of the Reynolds number \( Re \).

Recent numerical and theoretical studies have shown that the drag reduction by SHSs could extend to the turbulent flow region. Early simulations by Min & Kim (2004) model the SHS using slip boundary conditions with prescribed values of \( \lambda \), without resolving the wall topography. They show that \( DR \) increases from 0 to 29\% with increasing magnitudes of \( \lambda^* = \lambda/\delta \), from 0.04 to 3.6. Here, \( \delta \) is the viscous length scale defined as \( u/\nu \), the ratio between kinematic viscosity and friction velocity. The same model has been applied in later studies by Fukagata et al. (2006), Busse & Sandham (2012), and others. Fukagata et al. (2006) establish a theoretical relationship between \( \lambda^* \) and \( DR \) based on the upward shift of entire mean velocity profile. This relationship is later modified by Busse & Sandham (2012) using less free parameters. In another model (Martell et al. 2009, 2010), the SHS is represented by a combination of no-slip and shear free boundaries at the solid-liquid and air-liquid interfaces, respectively. Since this model resolves the SHS topography, the effects of \( \Phi_s, \beta \), and roughness type (post or ridge) on \( \lambda \) and drag reduction can be evaluated (Hasegawa et al. 2011, Jelly et al. 2014, Lee et al. 2015). Using streamwise grooves, Park et al. (2013) investigate a wide range of \( \Phi_s \), from 0.06 to 0.5, and \( \beta \), from 0.01\( \delta \) to 3\( \delta \), where \( \delta \) denotes the boundary layer thickness or half channel height. They show that the drag reduction increases with increasing \( Re \) or \( \beta \) or decreasing \( \Phi_s \). Moreover, they find a correlation between drag reduction and \( \lambda^* \) for different values of \( Re, \Phi_s \) and \( \beta \). See & Mani (2016) have recently reported a scale relation \( \lambda^*=(\beta^{+\lambda})/\Phi_s^{0.5} \) for an SHS with a post geometry.

Many experimental studies have confirmed that SHS reduces drag in turbulent flows using several methods to determine the skin-friction. Park et al. (2013b) utilize a strain gage connected a floating SHS suspended in a turbulent boundary layer. Srinivasan et al. (2015) perform tests in a Taylor-Couette facility and measure the torque on the SHS-covered inner rotor. Jung & Bhushan (2010) examine the drag reduction based on the pressure drop in a channel flow. Particle Image Velocimetry (PIV) based studies that resolve the flow at \( \gamma>5\delta_s \), estimate the skin-friction by fitting the mean velocity profile in the log region (Tian et al. 2015) or by linearly extending the total stress profile to the wall (Woolford et al. 2009). Recently, in Ling et al. (2016), the skin-friction is determined directly from the total stress at the top of the SHS by fully resolving the flow very close to the wall using digital holographic microscopy. Consistent with the numerical studies, the experimental results also show that \( DR \) increases with increasing \( Re \) or \( \beta \) and decreasing \( \Phi_s \). Using SHSs with streamwise grooves with \( \beta=60 \) and 120 \( \mu m \), Daniello et al. (2009) show that the SHS with the larger wavelength has a higher drag reduction. In addition, they show an increase of \( DR \) by up to 50\% as the \( Re \) increases from 2000 to 8000 (based on channel height and mean flow speed). For SHSs with streamwise grooves of various \( \Phi_s \), Park et al. (2014) show that \( DR \) increases by up to 75\% as \( \Phi_s \) decreases to 5\%. Using a randomly distributed rough SHS (spray-coated), Srinivasan et al. (2015) show that the drag reduction increases by up to 22\% as \( Re \) increases to 80,000. Moreover, they find a scale relation of \( \lambda^* \sim Re^{0.2} \) in the limit of high \( Re \). Ling et al. (2016) show good agreements between the measured drag reduction and the theoretically predicted levels (Busse & Sandham 2012, Fukagata et al. 2006) for SHSs involving both spanwise and streamwise slips.

In contrast, several other experimental studies involving randomly textured SHSs show that \( DR \) decreases with increasing \( Re \) (Aljallil et al. 2013, Bidkar et al. 2014, Henoch et al. 2006, Ling et al. 2016, Watanabe et al. 1999, Zhao et al. 2007). In a 6 mm pipe flow, Watanabe et al. (1999) show that a SHS with roughness height of less than 10 \( \mu m \) reduces the drag by 14\% for laminar flow, but does not reduce the drag in the turbulent flow regime. In a towing tank, Aljallil et al. (2013) show that a nanoscale textured SHS reduces the drag by about 30\% in the transition regime, but causes an increase of drag in fully turbulence regime. They attribute this trend to entrainment of air layer at high \( Re \). Bidkar et al. (2014) report that drag reduction seen at low \( Re \) diminishes with increasing \( Re \) when the surface roughness height \( k \) becomes comparable to \( \delta_s \), i.e., \( k^* = k/\delta_s > 0.5 \). In agreement with the latter, Ling et al. (2016) show that drag reduction diminishes for \( k^* \sim 1.0 \), and the skin friction increases for \( k^* \sim 3 \). Moreover, they show that with increasing \( k^* \) the wall stress transitions from drag reduction, when the viscous stress is the primary contributor to the total stress, to drag increase when the Reynolds shear stress is the primary contributor. In other studies by Peguero & Breuer (2009) and Greidanus et al. (2011), no significant drag
reduction is observed. Possible reasons may include air layer depletion, air layer vibrations, dominance of wall roughness effects, as well as measurement uncertainties and errors.

The SHSs have also been found to modify the near wall turbulence. Numerical simulations of patterned SHSs by Jelly et al. (2014), Min & Kim (2004), and Park et al. (2013a) have shown that the peak Reynolds stresses are reduced and the near-wall streamwise vortices are weakened. PIV measurements for a randomly roughed SHS by Vajdi Hokmabad & Ghaemi (2016) show that SHS suppresses the sweep and ejection events, increases the spanwise spacing of the low and high speed streaks, and attenuates the vortical structures in the buffer layer. On the contrary, simulations by Martell et al. (2010) show that the patterned SHS only shift the turbulent structures closer to the wall without modifying them significantly. The measurements on randomly roughed SHSs by Ling et al. (2016) also show that the peaks of normal Reynolds stresses move closer to the wall.

In summary, both numerical simulations, and a number of prior experiments, have shown a great promise for applying SHSs for turbulent drag reduction. However, there are only a few works, especially experimental, focusing on turbulent structures over the SHSs. There are also disagreements among the numerical studies about how the SHSs modify the near wall turbulence. Many questions still exist on how modifications to the structure of turbulence above the SHSs affect the wall friction. The present study expands the scope of the experimental effort described in Ling et al. (2016). We investigate the structure of mean flow and turbulence in the inner part of turbulent boundary layers over SHSs and compare them to those measured over smooth walls. This paper examines how the SHSs alter the turbulent quantities. The analysis is based on high resolution near wall velocity measurements using inline digital holographic microscopy (DHM). The slip velocity and local wall friction are calculated directly from the mean velocity and total stress at the top of the SHSs roughness. In addition to examination of Reynolds stress profiles, the data analysis examines the effects of SHSs on the eddy viscosity, Prandtl’s mixing length, and spatial energy spectra.

EXPERIMENTAL APPROACH

Water Tunnel and Super-Hydrophobic Surfaces

The experiments have been performed in a small, high-speed water tunnel described in Gopalan & Katz (2000) and Liu & Katz (2006). The flow is driven by two 15 HP (maximum) centrifugal pumps located 5 m below the test section, and passes through a settling tank, an electromagnetic flow meter, a settling chamber containing honeycombs and screens, as well as a 9:1 contraction before entering the test section. The transparent test section is 406 mm long, 61 mm high and 50 mm wide, as shown in Figure 1. The mean tunnel speeds (flow rate divided by the tunnel cross section), $U_m$, can be varied between 2 to 20 m/s. At the entrance to the test section, the bottom window contains a series of machined spanwise tripping grooves, which are located 165 mm (9–22 boundary layer heights) upstream of the SHS. The purpose of these grooves is to force early boundary layer transition to turbulence, as shown in prior studies (Liu & Katz 2013). The 152 mm long and 50 mm wide SHS is flush mounted on the same wall. The coordinate system is also shown, with $x$, $y$, and $z$ denoting the streamwise, wall-normal, and spanwise directions, respectively, and $x=0$ coinciding with the leading edge of the SHS.

One smooth surface and five SHSs, denoted as Smooth, SP-Por1, SP-Por2, SP-Al, ETH-Al and SB-Al, are included in the present discussion. The method of creating the textured surface and making it hydrophobic for each of these cases are listed in Table 1. The smooth surface is created on a PVC base and serves as the baseline. The five SHSs are created on either porous stainless steel bases (SP-Por1 and SP-Por2) or non-porous aluminum bases (SP-Al, ETH-Al and SB-Al). The purpose of using porous bases is to provide a mean for continuously replenishing the micro-air pockets as are entrained by the flow. They have permeability of 0.27–0.98 μm² and porosity of 17–26%, as specified by the manufacturers (Mott). During measurements, for the cases included here, the pressure below the porous wall is set to be slightly higher (by 300 Pa) than that in the test section. For the solid bases, the gauge pressure in the test section is set between 4 to 6 kPa. Hence, the air plastron is only mildly suppressed.

Several roughness manufacturing methods and different low surface energy molecules are applied
to generate the SHSs. For SP-Por1, SP-Por2 and SP-Al, the roughness is created by spray-coating (Srinivasan et al. 2011). The sprayed material for SP-Por1 and SP-Por2 is a mixture of poly methyl methacrylate (PMMA) binder and fluorinated polyhedral oligomeric silsesquioxane (F-POSS). These two surfaces are identical except for the roughness height, as discussed later. The sprayed material for SP-Al is a mixture of F-POSS and ethyl cyanoacrylate (superglue). For the ETH-Al, the roughness is created by etching the aluminum in 2.5 molar HCl for 20 minutes (Yang et al. 2011). For the SB-Al, the aluminum base is sandblasted first using sandpaper (grit 150) to create micro- pores and then etched in 12 molar HCl for 25 seconds to generate nanoscale structures (Pillutla et al. 2016). Both ETH-Al and SB-Al are further coated with low surface energy materials, as specified in Table 1, using chemical vapor deposition. Scanning electron micrograph (SEM) images of the porous base of SP-Por1 and the SHSs are provided in Figure 2(a)-(e).

The SHS topography is characterized by imaging the roughness elements using DHM at a resolutions of 0.68 μm/pixel in the x and y direction and ~100 μm in z direction. The holograms are recorded while the facility is filled with water, accounting for the presence of the air layer. By selecting a threshold of intensity to include roughness elements, which are nearly in focus, the projection local roughness height \( k(x) \) is tracked. Figure 3(a) shows a sample \( k(x) \) for SH-Por1. The corresponding probability density and cumulative distribution of \( k \) are shown in Figure 3(b). We characterize the roughness height using the root-mean-square (rms) value of \( k(x) \), and denote it as \( k_{\text{rms}}=\left(\int k^2 \, dx\right)^{0.5}/L \), where \( L \) is the sample length. The values of \( k_{\text{rms}} \) calculated from this method are listed in Table 1. The uncertainty, evaluated by selecting different intensity thresholds to estimate \( k_{\text{rms}} \), is less than 1 μm. Defining the origin of the vertical axis as the mean roughness height, the top of roughness, where the wall friction is evaluated, is selected to be located at \( y=2k_{\text{rms}} \). As shown by Figure 3(a) and (b), 95% of the roughness peaks are located below this elevation. When the roughness height is evaluated in air using a laser interferometer with 0.1 μm resolution, the values of \( k_{\text{rms}} \) are 10–20% lower than those obtained using DHM. Hence, results are consistent.

**Velocity Measurements**

A dual-view in-line DHM illustrated in Figure 4 is used to fully resolve the inner part of the boundary layer. The purpose of using two views is to separate between the reconstructed real and virtual (twin) images of the particles on both sides of the hologram plane, as discussed in detail in Ling & Katz (2014). The light source is an Nd-YAG laser (532 nm). Since very little energy is required for inline DHM, only light reflected from an uncoated flat glass surface is sufficient for acquiring the holograms. The beam is spatially filtered, expanded, collimated, and illuminates the beam, and spatially filtered, expanded, collimated, and illuminates

### Table 1: Specifications of five SHSs involved in this study, including base type, roughness manufacture method, surface chemistry, and rms roughness height. F-POSS denotes fluorinated polyhedral oligomeric silsesquioxane, F-silane as (heptadecafluoro-1,1,2,2-tetrahydrodecyl) trichlorosilane, and PTFE as polytetrafluoroethylene.

<table>
<thead>
<tr>
<th>#</th>
<th>Base</th>
<th>Application method</th>
<th>Chemistry</th>
<th>( k_{\text{rms}} ) μm</th>
</tr>
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<tr>
<td>Smooth</td>
<td>Solid</td>
<td>-</td>
<td>-</td>
<td>&lt;1</td>
</tr>
<tr>
<td>SP-Por1</td>
<td>Porous</td>
<td>Sprayed</td>
<td>F-POSS</td>
<td>4.8</td>
</tr>
<tr>
<td>SP-Por2</td>
<td>Porous</td>
<td>Sprayed</td>
<td>F-POSS</td>
<td>13.8</td>
</tr>
<tr>
<td>SP-Al</td>
<td>Solid</td>
<td>Sprayed</td>
<td>F-POSS</td>
<td>7.4</td>
</tr>
<tr>
<td>ETH-Al</td>
<td>Solid</td>
<td>Etched</td>
<td>F-silane</td>
<td>10.9</td>
</tr>
<tr>
<td>SB-Al</td>
<td>Solid</td>
<td>Sandblasted &amp; etched</td>
<td>PTFE</td>
<td>&lt;3</td>
</tr>
</tbody>
</table>
a sample volume seeded locally with 2 μm silver-coated glass particles. The particles are injected at a speed of less than 0.08 \( U_m \) from 100 μm ports located ~100 mm (1000 injector diameters) upstream of the sample volume. An 8x, long focal distance microscope objective magnifies the images, and focuses the selected hologram planes (red lines) on two 4400x6600 pixels, 5.5 μm/pixel, interline transfer, CCD cameras (Imperx ICL-B6640). Their focal planes are separated by 50 μm, and their calibrated spatial resolutions are 0.677 and 0.685 μm/pixel. The center of the 4.4x2.4x3.2 mm³ \((x\times y \times z)\) sample volume is located at \( z=2 \text{ mm} \) and \( x=70 \text{ mm} \), the latter indicating the distance from the leading edge of the SHS. The complex particle field associated with either hologram is calculated iteratively by propagating the wave field between the two planes (Denis et al. 2005). Reconstruction is performed using the Rayleigh-Sommerfeld kernel (Katz & Sheng 2010) in a series of \( x\times y \) planes separated by 13 μm, each containing only real images. Following Sheng et al. (2008) and Talapatra & Katz (2013), particle segmentation and tracking are performed to obtain 6000-10,000 randomly distributed vectors from each hologram pair. First-order Taylor series expansion and singular value decomposition (Sheng et al. 2008) project these vectors onto structured vector fields with grid spacing of 12.5δ_x,1δ_x,25δ_y, (x×y×z) for \( U_m=2 \text{ m/s} \), and 25δ_x,1δ_x,25δ_y, (x×y×z) for \( U_m=6 \text{ m/s} \). The entire sample volume size is 420δ_x×230δ_x×305δ_y, for \( U_m=2 \text{ m/s} \) and 1000δ_x×540δ_y,×710δ_z, for \( U_m=6 \text{ m/s} \). More than 1000 hologram pairs are acquired and processed to provide ~1000 instantaneous velocity fields. The typical data acquisition time is about two hours. The mean flow quantities are obtained by ensemble-averaging locally for each grid point, and then spatially averaging in the \( x \) and \( z \) directions. Symbols \( u \) and \( v \) are used to denote the instantaneous streamwise and wall-normal velocity components. Spatially averaged mean velocities are denoted as \( \overline{U} \) and \( \overline{V} \), and spatially average velocity fluctuations are denoted as \( \overline{u'u'} \) and \( \overline{v'v'} \) and Reynolds shear stress as \( \overline{u'v'} \).

The spatially averaged viscous stress is calculated using \( \tau^v = \mu \overline{U|\partial U/\partial y} \), where \( \mu = 1 \times 10^3 \text{ kg/m/s} \) using velocity gradients estimated from 5 neighboring points. The spatially averaged Reynolds shear stress \( \tau^R = -\rho \overline{u'v'} \), where \( \rho = 1 \times 10^3 \text{ kg/m}^3 \). The total stress is \( \tau = \tau^v + \tau^R \). To minimize the potential effects of form drag, the wall friction \( \tau_w \) and slip velocity \( \overline{U_s} \) are calculated at \( y=2k_{mix}\). The slip length is calculated using \( \lambda = \mu \overline{U|/\tau^v} \), where \( \tau^v \) is the viscous stress at the same elevation. Logarithmic fitting to the mean velocity profile in the regions where values of \( \gamma\overline{U|/\partial y} \) are nearly constant provides another estimate for the wall friction, and denoted as \( \tau_w^{Log} \). The friction velocity is calculated using \( u_* = \left(\tau_w'/\rho\right)^{1/2} \).

Two-dimensional (2D) PIV at a lower resolution (5.4 μm/pixel) and a larger sample area (36×24 mm², \( x\times y \)) has also been used to obtain the entire boundary layer profile in the same areas as the center of the DHM volume. The instantaneous velocities are calculated using standard PIV cross-correlations (Roth & Katz 2001) with a window size of

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**Figure 3:** (a) A sample distribution of roughness height \( k(x)/k_{rms} \), and (b) the corresponding probability density (solid line) and cumulative distribution (dashed line) for the SP-Por1, \( k_{rms}=4.8 \mu m \). The dash-dot lines denote the location \( k/k_{rms}=2 \), where wall stresses are calculated.

**Figure 4:** The dual view in-line digital holographic microscopy setup for high resolution velocity measurements.
For \( U_m = 2 \text{ m/s} \) and \( 128 \delta_s \times 32 \delta_s \) (xy) for \( U_m = 6 \text{ m/s} \) with 50% overlap. Mean profiles are obtained by averaging ~50 realizations. The boundary layer thicknesses, \( \delta_{bu} \), are calculated based on the elevation where 99% of the maximum velocity \( (U_0) \) is reached.

In the rest of the paper, a superscript + is used for quantities that are normalized by \( u_\infty \) and \( \delta_{bu} \) as well as a subscript 0 for quantities measured above the smooth wall (the baseline). The combination of the superscript + and subscript 0 denotes quantities normalized by \( u_\infty \) and \( \delta_{bu} \). The magnitudes of the drag reduction are defined as \( DR = (\tau_{w0} - \tau_w)/\tau_{w0} \), where \( \tau_{w0} \) is the wall friction for a smooth wall at same \( Re \). uncertainties caused by the selection of top of the measurement are defined as \( \alpha \), \( \beta \), \( \gamma \), and \( \delta_{bu} \) are calculated. The larger of the two differences is selected as the uncertainty.

**RESULTS AND DISCUSSIONS**

Table 2 presents results from 9 tests with \( U_m \) ranging from 2.0 to 5.9 m/s, \( Re \) from 20,020 to 120,320 and \( Re \) from 700 to 4400. Three of them are for the baseline smooth walls, and six are for SHSs. The measured results, including \( \delta_{bu} \), \( \tau_w \), \( \tau_w^{Log} \), \( u_\infty \), \( \delta_{bu} \), \( U^* \), \( \lambda \), and \( DR \) are listed, along with their uncertainties. Table 2 also provides the values of static contact angles before and immediately after the experiments, denoted as \( CA^{bf} \) and \( CA^{af} \), respectively. Both are measured by placing a \( \sim4 \) mm (\( \sim270 \mu\text{L} \)) water droplet on top of the surface. Most SHSs show a slightly reduction of the contact angle, but remain super-hydrophobic after the experiments. For all cases including the one with the largest \( U_m \), the SHS surfaces appear to be very shiny when viewed from the side due to the total internal reflection from the air-water interfaces. While it does not guarantee that the entire surface is covered by the air layer, the uniformity of the reflection suggests that most of it does. Furthermore, high speed holographic movies confirm that the interface vibrates at all speeds during the measurement. The boundary layer thicknesses on the SHSs remain very similar to those on a smooth wall for the same \( U_m \), probably due to the relatively short length of the SHSs (\( \sim 17 \delta_{bu} \)). In addition, as discussed in Ling et al. (2016), differences between \( \tau_w \) and \( \tau_w^{Log} \) for all the current SHSs can be attributed to non-equilibrium conditions caused by the streamwise location of the measurements. The log regions have not fully adjusted to the new wall stresses, and values of \( \tau_w^{Log} \) are more consistent with the smooth wall results. Listed symbols in Table 1 are applied for all the figures shown in this paper.

Four SHSs satisfying 700<\( Re < 900 \) and \( k_{rms} < 1 \) show 10% to 30% reduction in wall friction compared to the smooth wall at. The corresponding slip velocities are 15% to 30% of \( U_m \) and the slip lengths vary from 30 to 110 \( \mu\text{m} \) (\( \lambda^{bf} < 10 \)). These values agree with the typically reported results obtained in microfluidic devices (e.g., Lee et al. 2008). For a surface manufactured in the same way as SB-Al, Pillutla et al. (2016) use a rheometer to measure a slip length of \( \lambda = 41 \mu\text{m} \), in agreement with the current value of \( \lambda = 48 \mu\text{m} \), in spite of the very different \( Re \). In Ling et al. (2016), we also report that the present data confirms

<table>
<thead>
<tr>
<th>Sample</th>
<th>( U_m ), m/s</th>
<th>( \delta_{bu} ), mm</th>
<th>( Re )</th>
<th>( k_{rms} )</th>
<th>( CA^{bf} ) (( \pm2^\circ ))</th>
<th>( CA^{af} ) (( \pm2^\circ ))</th>
<th>( \tau_w ), Pa</th>
<th>( \tau_w^{Log} ), Pa</th>
<th>( u_\infty ), m/s</th>
<th>( \delta_{bu} ), ( \mu\text{m} )</th>
<th>( \bar{U}^* ), m/s</th>
<th>( \lambda ), ( \mu\text{m} )</th>
<th>( DR )</th>
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<td>863</td>
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<td>-</td>
<td>-</td>
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<td>0.095</td>
<td>10.5</td>
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<td>148°</td>
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<td>161°</td>
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<td>&lt;0.24</td>
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<td>150°</td>
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<td>6.5</td>
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<td>7.7</td>
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Table 2: Experimental conditions and measurement results for all nine cases presented in this paper. The indicated symbols are used for all the figures shown in this paper.
the functional relationship between $\lambda^*$ and drag reduction introduced by Fukagata et al. (2006). In agreement with previous findings (Bidkar et al. 2014, Ling et al. 2016), the current results also show that drag reduction diminishes for $k_{rms}^*>1$. The same SP-Al which reduces drag at low $Re$ when $k_{rms}^*$=0.59 has nearly zero drag reduction for $Re$=1700 and $k_{rms}^*$=1.65. For $k_{rms}^*$=3.28 (SP-Por2), the SHS causes ~10% increase in wall friction where the roughness effect overcomes the SHSs effect.

**Mean Flow Quantities**

The present study expands the scope of the experimental effort described in Ling et al. (2016), which already discusses the mean flow quantities for the smooth walls and several SHSs (SP-Por1, EH-Al and SP-Por2). These data are mentioned briefly here, while focusing on the others. Figure 5(a) shows the profiles of viscous, Reynolds and total shear stresses for the four drag reduction SHS cases ($k_{rms}^*<1$) together with those of the corresponding smooth wall at same $U_m$ and $Re$. All profiles are normalized based on smooth wall values to highlight the differences. At $y^*<30$, all the SHSs have lower viscous stresses compared to those of the smooth wall. The Reynolds shear stress on the top of the roughness is nearly zero for $k_{rms}^*<0.5$ (SP-Por1 and SB-Al), but increases to $\sim0.2\tau_w$ for $0.5<k_{rms}^*<1$ (ETH-Al and SP-Al). At $y^*<10$, the Reynolds shear stresses of all the SHSs are slightly larger than those on the smooth wall. Except for SB-Al, the peak values of the Reynolds shear stresses appear to have a reverse trend, namely that they decrease with increasing drag reduction. Trends of the dimpled SB-Al (Figure 2e), which also has the smallest roughness height differ from others, e.g., having a lower total stress over the entire boundary layer.

Figure 5(b) shows the mean velocity profiles for the same five surfaces, scaled by their own $u_\tau$. As expected, all the drag reduction SHS profiles are shifted upward from that the smooth wall. Except for SB-Al, the upward shift in the log region is smaller than near wall region, a trend attributed by Min & Kim (2004) to effects of spanwise slip. For SB-Al, the shift is nearly nearly uniform, with unclear implications. Figure 5(c) shows the streamwise and wall-normal velocity fluctuations. The insert presents the same profiles normalized by the smooth wall parameters, allowing us to compare the actual magnitudes. Near the wall ($y^*<10$) and around the maxima, all the SHSs have higher normalized velocity fluctuations than those of the smooth wall. The possible reasons are roughness effects, monition of the air-water interface, and etc. However, the actual peak magnitudes are similar to those of the smooth wall. Consistent with the numerical simulation

**Figure 5:** Drag reduction cases: (a) Viscous, Reynolds and total shear stresses, normalized by the smooth wall inner units; (b) Mean velocity profiles scaled by their own $u_\tau$; and (c) Streamwise and wall-normal velocity fluctuations, also scaled by their own $u_\tau$, with the insert showing the same quantities scaled by the smooth wall units.
by Martell et al. (2010), the peak locations of $<u'u'>$ and $<v'v'>$ on the SHSs are closer to the wall compared to those on the smooth wall.

Figure 6(a) shows the profiles of viscous, Reynolds and total shear stresses for two SHSs without drag reduction and the two corresponding smooth walls, both normalized by the smooth wall parameters. Near the wall, the viscous stresses on both SHSs are much lower and the Reynolds stresses are much higher than those of the smooth walls. Both also have a broader constant stress layer, and elevated Reynolds stresses extending to the outer layer. Evidently, the Reynolds shear stress is the primary contributor to the total wall stress. Figure 6(b) shows the corresponding mean velocity profiles scaled by their own $u_\tau$. Very close to the wall ($y^+<10$), both SHSs have a higher momentum than those of the smooth walls. In the log region, the profile of the SHS with nearly zero drag reduction does not differ significantly from that of the smooth wall. The profile of the drag increase case is located below the smooth wall profile, consistent with trends of rough walls. The peaks of streamwise velocity fluctuations, shown in Figure 6(c), are broad and located very close to the top of the roughness elements. These trends are also consistent with those of rough wall boundary layers (Hong et al. 2011; Jimenez 2004). Accordingly, the magnitude of $<v'v'>$ for the drag increase case is also much larger than that of the smooth wall.

**Turbulent Kinetic Energy Shear Production**

The turbulent kinetic energy (TKE) shear production $P_s$ is calculated from $P_s = -<u'u'>d\overline{U'}dy$. Figure 7(a) shows the distributions of $P_s$ for all 9 cases. Following Marusic et al. (2010), the profile of $yP_s$ is also calculated and plotted in Figure 7(b). Here, the area under each profile in the semi-log plot is equals to the integral of $P_s$. The DNS result of Spalart (1988) are also included for comparison. All magnitudes are normalized based on the smooth wall inner units. The three smooth walls profiles, which represent different $Re_\tau$, share nearly the same peak value and location, and all agree with the DNS results. For the SHSs, at the top of roughness, the magnitude of $P_s$ increases with $k_{rms}$. It is nearly zero for $k_{rms}<0.5$, and high for $k_{rms}>1.0$. However, regardless of drag reduction or increase, at $y^+<5$, all the SHSs have higher production rates than those of the smooth walls. Except the dimpled SB-Al, the $P_s$ peaks of the SHSs are located closer to the wall than the smooth wall. High production rates near roughness peaks have been reported before both experimentally (Hong et al. 2011) and computationally (Ikeda & Durbin 2007). Two surfaces, namely SB-Al
and the $k_{rms}^* = 1.65$ SP-Al, have significantly lower peak magnitudes for unclear reasons.

Figure 7(b) shows that for all the drag reduction cases, the production rates in the log region are lower than that of the smooth walls. In contrast, the surfaces with $k_{rms}^* \geq 1$, the production rates are much higher than those of the smooth walls, and increase with elevation. Keeping in mind that the majority of the TKE in boundary layer is produced in the log region (Marusic et al. 2010). Figure 7(b) indicates that in spite of the higher Reynolds stresses very near the wall, the overall shear productions in drag-reducing SHSs are lower than that of the smooth wall.

**Eddy Viscosity and Prandtl’s Mixing Length**

Based on turbulent-viscosity hypothesis (Pope 2000):

$$\langle u'v' \rangle = v_T \left( \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \right),$$  \hspace{1cm} (1)

where $v_T$ is the so-called eddy viscosity. The second term on the right side of this equation is usually neglected in turbulent boundary layers, giving:

$$v_T \approx -\frac{\langle u'v' \rangle}{\partial U/\partial y}$$  \hspace{1cm} (2)

We also perform the analysis using the total stress to calculate the total viscosity $v_T'$:

$$v_T' = \frac{\tau_{i,j}\rho}{\partial U_j/\partial y} = v_T + v$$  \hspace{1cm} (3)

Accordingly, one can also calculate two values for the Prandtl’s mixing length, $l_m$ and $l_m'$:

$$l_m = \left( \frac{v_T}{\partial U/\partial y} \right)^{1/2} \quad \text{and} \quad l_m' = \left( \frac{v_T'}{\partial U/\partial y} \right)^{1/2}$$  \hspace{1cm} (4)

Figure 8(a) and (b) show the distributions of $v_T$ and $v_T'$, respectively, along with Spalart’s (1988) DNS results. The model by Van Driest (1956):

$$l_m^* = \kappa y^* \left( 1 - e^{-\gamma/26} \right)$$  \hspace{1cm} (5)

is included in Figure 8(b). All values are normalized based on their own inner units. For the current measurement domains ($0 < y < 0.3 \delta_90$), both $v_T$ and $l_m$ increase with $y$. However, at higher elevations, the values of $v_T$ are expected to decrease with $y$, while the mixing length is expected to plateau. All the current smooth wall results nearly collapse onto the DNS data and Van Driest model at $y^* < 100$. At $y^* < 10$ where viscous stress dominates, the values of $v_T$ and $l_m^*$ are smaller than 1. At the beginning of log region $y^* > 30$, $v_T/\delta_90$ and $l_m^*$ are larger than 5. The smooth wall results do not collapse at $y^* > 100$, presumably due to Reynolds number dependent changes to turbulence in the outer layer (Smits et al. 2011).

For the SHSs, both $v_T$ and $l_m$ increase with elevation. Near the wall ($y^* < 10$), they are higher than those of smooth walls, increasing from values of less than 1 for drag reduction cases, to values larger than 1 for the drag increase surfaces. These trends correspond to the increase of Reynolds shear stress and reduction in viscous stress with increasing $k_{rms}^*$. For the two drag increase cases, the values collapse. The differences between all the SHSs and the smooth walls results diminish with increasing $y$, and they appear to coincide.
in the log layer. The profiles of \( l_m^+ \) collapse better than those of \( v_f/\tau_0 \).

The inserts in Figure 8(a) and (b) show the distributions of \( v_f/\tau_0 \) and \( v_f/\tau_0 \) only differ by 1, the trends are inherently similar. However, being presented in a log-log plots, near the wall (\( y^+ < 10 \)), the profiles of \( v_f/\tau_0 \) and \( l_m^+ \) for the drag reduction cases appear to be much closer to those of the smooth wall. Conversely, results for the cases where roughness begins to dominate, values of \( v_f/\tau_0 \) and \( l_m^+ \) are much higher than those of smooth walls.

**Spatial Energy Spectra**

Fast Fourier transform (FFT) of the instantaneous 2D-PIV data is used for calculating the spatial energy spectra, \( E_{11}(k_x) \) and \( E_{22}(k_x) \), of the streamwise and wall-normal velocity fluctuations, respectively, with \( k_x \) being the spatial wavenumber. Each realization contains ~100 vectors spaced by \( 32\delta_v \) in the \( x \) direction. Details of the procedures are provided in Hong et al. (2011). The resolved wavenumber falls in the range of \( 3 \times 10^{-4} < k_x \delta_v < 0.02 \). Results have been obtained for two drag reduction SHSs (SP-Por1 and SP-Al) and the corresponding smooth wall. Figure 9(a) and (b) show the spectra at \( y^+_0 = 30 \) and \( y^+_0 = 100 \), respectively. The spectral values are normalized by \( u^+_0 \delta_v \) and the wavenumber is normalized by \( \delta_v \). Several trends are evident, as some expected for turbulent boundary layers. For example, the difference between \( E_{11}(k_x) \) and \( E_{22}(k_x) \), decreases with elevation, and with increasing \( k_x \). This trend has been attributed to the suppression of large scale wall-normal turbulence near the wall. Second, the slopes increase with wavenumber, as \( E_{11} \) transitions from the
turbulence production range to the inertial subrange (slope $-5/3$), and then to the dissipation range.

At $y^*_0 = 30$, the values of $E_{11}$ for the SHSs are similar to those of the smooth wall at low wavenumbers ($k_0 \delta_{0,w} < 0.001$), but are clearly smaller at higher wavenumbers. Conversely, the magnitudes of $E_{22}$ of both SHSs are larger than those of the smooth wall at low wavenumbers $k_0 \delta_{0,w} < 0.001$, but are similar at high wavenumbers. These trends suggest that the SHSs alter the structure of the near wall turbulence. While the rough and/or vibrating SHSs are expected to increase the wall normal fluctuations, consistent with the $< v' v' >_0$ profiles, the reasons for the slight suppression of small-scale streamwise energy is not clear. In a recent paper, Vajdi Hokmabad & Ghaemi (2016) report that the ejection and sweeping events are suppressed and the width of low speed streaks increases above the SHS, which would presumably cause a decrease of streamwise fluctuations in inertial range. The differences in $E_{22}$ spectra between the smooth wall and the SHSs diminish at $y^*_0 = 100$. The differences in small scale streamwise fluctuations persist.

CONCLUSIONS

The current study combines high resolution digital holographic microscopy (DHM) and particle image velocimetry to characterize the flow structure and turbulence in the inner part of turbulent boundary layers over several super-hydrophobic surfaces. The surface textures are generated using various methods, including spraying, etching and sandblasting. The rms roughness height $k_{rms}$ is in the range of 2$\leq k_{rms} \leq 14$ μm. The Reynolds numbers are 20,020$\leq Re_\tau \leq 120,320$, and 700$ \leq Re \leq 4400$. The viscous length scale falls in the 4$\leq \delta \leq 12$ μm range, hence the normalized roughness height are 0.2$ \leq k_{rms}^* \leq 3.3$. The slip velocity and local wall friction are calculated directly from the mean velocity gradients and Reynolds shear stress at the top of SHSs. The results are compared to smooth wall mean flow and Reynolds stress profiles obtained in the same facility.

For $k_{rms}^* < 1$, the SHSs have lower viscous stress and nearly zero Reynolds shear stress at the top of the SHSs, resulting in 10% to 30% reduction of wall stress. The slip velocities vary from 15% to 30% of the mean flow speed. The slip lengths are in the range of 3$\leq l^* \leq 10$, agreeing with previously reported values measured in microfluidic devices. For some surfaces, the drag reduction diminishes with increasing $k_{rms}^*$ (increasing $Re_\tau$), consistent to previous studies (Bidkar et al. 2014, Ling et al. 2016). At $k_{rms}^* > 3$, the near wall stresses are dominated by Reynolds shear stress and the SHS causes an increase of drag. The near wall mean velocity profiles show a transition from momentum excess for $k_{rms}^* < 1$ to momentum deficit for $k_{rms}^* > 1$. The SHSs also change the profiles of longitudinal and shear Reynolds stresses across the boundary layer. The most notable outcome is a decrease of shear production rates over the drag reducing SHSs in the log region, and an increase of production rates when the roughness effects take over. Irrespective of roughness height, the eddy viscosity and Prandtl’s mixing length close to the wall ($y^* < 10$) above the SHSs are substantially higher than those of the smooth wall. These trends are a direct result of the increase in Reynolds shear stresses and decrease in mean velocity gradients (viscous stresses) near the wall. The SHSs, which cause drag increase, have particularly high values. In the log region, the SHS profiles of eddy viscosity and Prandtl’s mixing length coincide with those of the smooth wall although trends/slopes differ. Energy spectra clearly show that the drag-reducing SHSs alter the structure of near wall turbulence, reducing the inertial range streamwise fluctuations, and increasing the wall normal large scale fluctuations. The latter effect diminishes with increasing distance from the wall, but the former trend persists. Near future research will probe the origin of these phenomena.

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